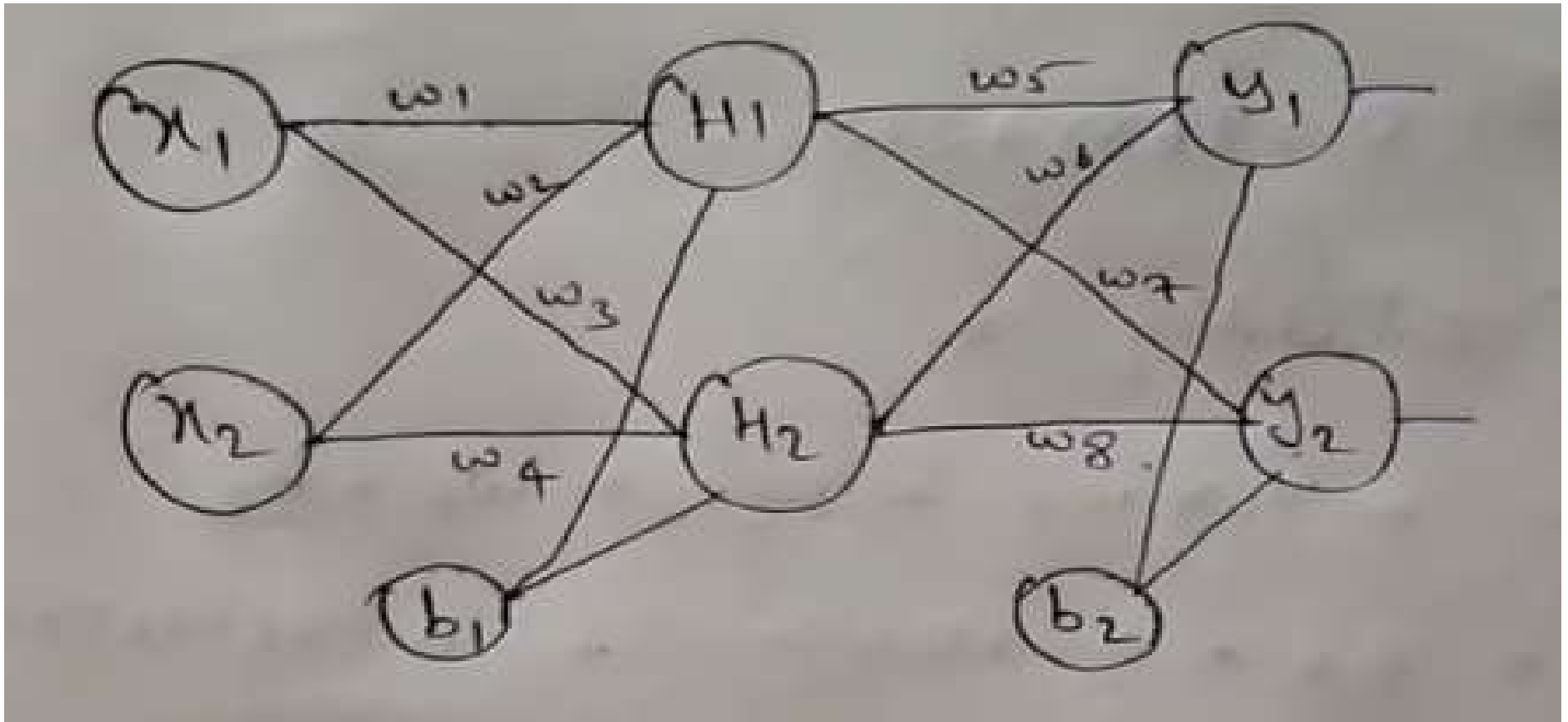


Back Propagation Network (BPN)

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Simple Structure of BPN



Example

$$x_1 = 0.05$$

$$x_2 = 0.10$$

$$b_1 = 0.35$$

$$b_2 = 0.60$$

Target values

T_1 T_2

0.01 0.99

Initial weights

$$w_1 = 0.15$$

$$w_2 = 0.20$$

$$w_3 = 0.25$$

$$w_4 = 0.30$$

$$w_5 = 0.40$$

$$w_6 = 0.45$$

$$w_7 = 0.50$$

$$w_8 = 0.55$$

Forward Pass

$$H_1 = x_1 * w_1 + x_2 * w_2 + b_1$$

$$= 0.05 * 0.15 + 0.10 * 0.20 + 0.35$$

$$= 0.3775$$

$$\text{out } H_1 = \frac{1}{1 + e^{-H_1}} = \frac{1}{1 + e^{-0.3775}} = 0.593269992$$

Now for calculating y_1

$$y_1 = \text{out } H_1 * w_5 + \text{out } H_2 * w_6 + b_2$$

$$= 0.4 * 0.593269992 + 0.596884378 * 0.46 + 0.6$$

$$= 1.105905967$$

$$\text{out } y_1 = \frac{1}{1 + e^{-y_1}} = \frac{1}{1 + e^{-1.105905967}} = 0.75136507$$

In the same way

$$\text{out } y_2 = 0.772928465$$

Calculating Total Error

$$E_{\text{Total}} = \sum \frac{1}{2} (\text{target} - \text{output})^2$$

$$= \underbrace{\frac{1}{2} (T_1 - \text{out } y_1)^2}_{E_1} + \underbrace{\frac{1}{2} (T_2 - \text{out } y_2)^2}_{E_2}$$

$$= \frac{1}{2} (0.01 - 0.75136507)^2 + \frac{1}{2} (0.99 - 0.792)^2$$

Backward Pass

To update weights.

Consider w_5

$$\text{Error of } w_5 = \frac{\partial E_{\text{total}}}{\partial w_5}$$

$$\frac{\partial E_{\text{total}}}{\partial w_5} = \frac{\partial E_{\text{total}}}{\partial \text{out } y_1} * \frac{\partial \text{out } y_1}{\partial y_1} * \frac{\partial y_1}{\partial w_5}$$

$$\frac{\partial E_{\text{total}}}{\partial \omega_5} = \frac{\partial E_{\text{total}}}{\partial \text{out } y_1} * \frac{\partial \text{out } y_1}{\partial y_1} * \frac{\partial y_1}{\partial \omega_5}$$

$$E_{\text{total}} = \frac{1}{2} (T_1 - \text{out } y_1)^2 + \frac{1}{2} (T_2 - \text{out } y_2)^2$$

$$\frac{\partial E_{\text{total}}}{\partial \text{out } y_1} = 2 * \frac{1}{2} (T_1 - \text{out } y_1)^{2-1} * -1 + 0$$

$$= - (T_1 - \text{out } y_1)$$

$$= - (0.01 - 0.75136507)$$

$$\frac{\partial \mathcal{L}_{\text{total}}}{\partial \text{out } y_1} = 0.74136507$$

$$\text{out } y_1 = \frac{1}{1 + e^{-y_1}}$$

$$\frac{\partial y_1}{\partial w_5} = 0.593269992$$

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial out y_1} * \frac{\partial out y_1}{\partial y_1} * \frac{\partial y_1}{\partial w_5}$$

$$= 0.74136507 * 0.186815602 * 0.593269992$$

$$\frac{\partial E_{total}}{\partial w_5} = 0.082167041 \rightarrow \text{change in } w_5$$

Updating w_5

$$w_5 = w_5 - \eta * \frac{\partial E_{total}}{\partial w_5}$$

$$= 0.4 - 0.5 * 0.082167041$$

$$w_5 = 0.35891648$$

$\eta \rightarrow$ learning rate.
 $= 0.5$

in the same way.

$$\omega_6 = 0.408666186$$

$$\omega_7 = 0.511301270$$

$$\omega_8 = 0.561370121$$

$$\frac{\partial E_{\text{total}}}{\partial \omega_1} = \frac{\partial E_{\text{total}}}{\partial \text{out} H_1} * \frac{\partial \text{out} H_1}{\partial H_1} * \frac{\partial H_1}{\partial \omega_1}$$



$$\frac{\partial E_{\text{total}}}{\partial \text{out} H_1} = \frac{\partial E_1}{\partial \text{out} H_1} + \frac{\partial E_2}{\partial \text{out} H_1}$$

$$\frac{\partial \epsilon_1}{\partial y_1} = \frac{\partial \epsilon_1}{\partial \text{out } y_1} * \frac{\partial \text{out } y_1}{\partial y_1}$$

$$= 0.74136507 * 0.186815602$$

$$= 0.138498562$$

$$\frac{\partial y_1}{\partial \text{out}_{H1}} = w_5 = 0.40.$$

$$\begin{aligned} \frac{\partial E_1}{\partial \text{out}_{H1}} &= 0.138998562 * 0.40 \\ &= 0.055599425 \end{aligned}$$

$$\frac{\partial E_2}{\partial \text{out}_{H1}} = -0.019049119.$$

$$\frac{\partial E_{\text{total}}}{\partial w_1} = \frac{\partial E_{\text{total}}}{\partial \text{out } H_1} * \frac{\partial \text{out } H_1}{\partial H_1} * \frac{\partial H_1}{\partial w_1}$$

$$= 0.03635 * 0.241300 * 0.05$$

$$\frac{\partial E_{\text{total}}}{\partial w_1} = 0.000438569$$

$$\omega_1 = \omega_1 - \eta * \frac{\partial E_{total}}{\partial \omega_1}$$

$$= 0.15 - 0.5 * 0.000438$$

$$\omega_1 = 0.149780716$$

$$\omega_1 = 0.149780716.$$

In the same way.

$$\omega_2 = 0.19956143$$

$$\omega_3 = 0.24935114$$

$$\omega_4 = 0.299150229.$$